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Group Analysis of Masses and Spins in Curved Space-Time: Cosmological and Experimental Consequences

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Abstract

Recent developments in spontaneously broken gauge theories as well as in group analysis of masses and spins in curved space-time indicate that rest masses may change as a function of cosmic time. Such as effect is incompatible with standard cosmological models. A set of cosmological models that incorporate mass variation is introduced. These cosmological models are shown to be fully compatible with the group analysis, yielding exactly the same formula; they are used therefore as a theoretical testing ground for the hypothesis of mass variation. The following consequences of this hypothesis are obtained: (1) Cosmological red-shifts are shown to correspond to a contracting, rather than expanding, universe. (2) The effects of mass variation on planetary orbits are calculated; they are not precluded by the data. Conclusive experimental evidence is expected within a few years.

1. *Introduction*

For space-time with constant four-dimensional curvature the theory of group representations reveals a deep relationship between the geometry of the universe and fundamental properties of particles: Eigenvalues of the Casimir operators of the group of motion correspond to masses and spins (Wigner, 1939; Thomas, 1941).[1] Realistic cosmological models, however, have a four-dimensional curvature $K(t)$ which is a function of cosmic time t. Such models do not have a group of motion.[2] A relationship between the geometry of the model and fundamental properties of particles was nevertheless derived (Malin,

[1] Thomas' work was corrected by Newton (1950). For an introduction to the groups of motion of space-times with constant four-dimensional curvature, which contains further references, see Gürsey (1965).

[2] For realistic cosmological models groups of motion have to be replaced by the more general structure of quasigroups; see Halpern & Malin (1971, 1974).

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1974), and it led to the following result: While spins of particles are rigorously conserved, all masses change as a function of cosmic time,[3] decrease in an expanding universe, and increase in a contracting one.

For the standard cosmological models of general relativity, rest masses are conserved. One needs, therefore, a different set of cosmological models to serve as a theoretical testing ground for the hypothesis of mass variation. Such a set can be obtained from the equations

$$R_{\mu\nu} = -4\pi\kappa T_{\mu\nu} \tag{1.1}$$

(Malin, 1975).[4,5] Equations (1.1) cannot, in general, replace Einstein's equations, because their hydrodynamic consequences are inconsistent with experimental results (Lindblom & Nester, 1975). The cosmological models that are derived from equation (1.1) for isotropic, spatially homogeneous universes, however, are not precluded by observational and experimental data and incorporate time variation of all masses. They can serve therefore as a theoretical testing ground for the hypothesis of mass variation.

In the present paper these cosmological models and the group analysis of masses and spins in curved space-time are shown to yield exactly the same result for mass variation. We then proceed to derive consequences of such a mass variation for the interpretation of the cosmological red-shift and for planetary orbits.

Section 2 presents the cosmological models of equation (1.1). Previous results are summarized and some new relationships are obtained. All the models are shown (a) to have positive four-dimensional curvature K (as compared with the standard cosmological models of general relativity, all having $K < 0$); (b) to imply a time variation of all masses m according to the formula

$$m(t) \sim [K(t)]^{1/2} \tag{1.2}$$

In Sections 3-5 the method of group analysis of masses and spins (Malin, 1974) is applied to cosmological models with $K > 0$. The relevant group is $O(3, 2)$, as compared with $O(4, 1)$ for the case $K < 0$. As pointed out by Dirac (1935) unresolved difficulties in formulating quantum theory in space-times

[3] Recently the possibility of mass variation was also raised in the context of Salam's (1968) and Weinberg's (1967) spontaneously broken gauge theories of weak and electromagnetic interactions. Spontaneously broken gauge theories were reviewed by Mahanthappa (1973) and Abers & Lee (1973). Consequences of such theories for the gravitational field equations were discussed by Linde (1974), Dreitlein (1974) and Veltman (1974). In the context of such theories the energy of the vacuum depends on the temperature of the medium and therefore varies on the cosmological scale (Kirzhnitz, 1972; Kirzhnitz & Linde, 1972; Wienberg, 1974). Various rest masses may vanish to zeroth order and can then be calculated as higher-order effects (Weinberg, 1972a; Georgi & Glashow, 1972). Such effects may change with the temperature of the medium and therefore change with time.

[4] $R_{\mu\nu}$ is the Ricci tensor, R is the Riemann scalar, κ is the gravitational constant, and $T_{\mu\nu}$ is the energy-momentum tensor.

[5] The constant $4\pi\kappa$ in equation (1.1) was derived by the requirement that Newton's law of gravitation is obtained as a limiting case, in complete analogy with the derivation of the constant $8\pi\kappa$ in Einstein's equations.

of constant *negative* curvature, with $O(4, 1)$ as the group of motion, do not occur in space-time of constant *positive* curvature, with $O(3, 2)$ as the group of motion. The group analysis yields equation (1.2) for all isotropic, spatially homogeneous space-times with $K(t) > 0$; the consistency of the group approach with the cosmological models of equation (1.1) is thus established.

Section 6 is devoted to the cosmological red-shifts. Since mass variation involves a variation of all spectroscopic frequencies, the observed red-shift is due to a combination of two effects: (a) The change in the frequency of a free photon as it travels from a distant galaxy; (b) the concurrent change in the frequency of the atomic transition corresponding to the emitted photon. Both shifts are towards the red in an expanding universe and towards the blue in a contracting one; and since (b) is shown to be stronger than (a), an observed *red*-shift corresponds to a *contracting* universe. In the context of equation (1.1) available evidence does not preclude a universe, which is at present in a contracting phase.

An estimate for the rate of change of masses is obtained in Section 6. Consequences for planetary motion are derived and compared with available evidence. Conclusive results are not available yet, but are expected within the next few years. Time variation of all masses will produce a variety of other effects. Beta-decay experiments as well as astronomical, astrophysical and geological evidence were discussed in a previous article (Malin, 1974). Following F. J. Dyson's analysis of evidence for time variation of fundamental constants (Dyson, 1972), it was shown that available data do not preclude such a mass variation.

2. *Cosmological Models*

If the universe is assumed to be isotropic, spatially homogeneous, and filled with a congruence of fundamental world lines, then it is possible to choose a canonical coordinate system (t, x_1, x_2, x_3), such that the metric tensor in the coordinate system takes the Robertson–Walker form

$$ds^2 = dt^2 - S^2(t)(dx_1{}^2 + dx_2{}^2 + dx_3{}^2)/(1 + \tfrac{1}{2}kr^2)^2 \tag{2.1}$$

where

$$r = (x_1{}^2 + x_2{}^2 + x_3{}^2)^{1/2} \tag{2.2}$$

and on the fundamental world lines $ds^2 = dt^2$. The coordinate t in the canonical frame of reference is called "cosmic time" and the function $S(t)$ in equation (2.1) is called "the expansion function."

For the purpose of calculating and discussing cosmological models the energy-momentum tensor will be assumed, as usual, to have the form

$$T_\nu{}^\mu = \begin{pmatrix} \rho(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.3}$$

where $\rho(t)$ is the matter density. The standard cosmological models of general relativity are then obtained from the following equations for the expansion function $S(t)$:

$$8\pi\kappa\rho = 3\dot{S}^2/S^2 + 3k/S^2 \tag{2.4a}$$

$$2S\ddot{S} + \dot{S}^2 + k = 0 \tag{2.4b}$$

The cosmological models of equation (1.1), however, are obtained from the following equations (Malin, 1975):

$$4\pi\kappa\rho = -3\ddot{S}/S \tag{2.5a}$$

$$S\ddot{S} + 2\dot{S}^2 + 2k = 0 \tag{2.5b}$$

where in equations (2.4) and (2.5) $k = +1, 0, -1$ for spherical, Euclidean, and pseudospherical spaces, respectively.

The four-dimensional curvature of cosmological models is, in general, a function of cosmic time t and is given in terms of the expansion function $S(t)$ by

$$K(t) = -(1/12)g^{\alpha\beta}R_{\alpha\beta} = -(1/2S^2)(S\ddot{S} + \dot{S}^2 + k) \tag{2.6}$$

It follows from equations (2.4) and (2.5) that for all the cosmological models of Einstein's equations $K(t)$ is negative at all times:

$$K(t) = -\tfrac{2}{3}\pi\kappa\rho(t) < 0 \tag{2.7}$$

and for the cosmological models of equation (1.1), $K(t)$ is always positive:

$$K(t) = +\tfrac{1}{3}\pi\kappa\rho(t) > 0 \tag{2.8}$$

The spherical, Euclidean, and pseudospherical space solutions of equation (2.4) are well known and will not be reproduced here.[6] Equation (2.5) also allows for spherical, Euclidean, and pseudospherical space solutions, which are given, up to a final quadrature, by the following equation:

$$t = \int_0^S (DS^{-4} - k)^{-1/2}\, dS \tag{2.9}$$

where D is a constant. The first- and second-order derivatives of $S(t)$ are given by

$$dS/dt = (DS^{-4} - k)^{1/2} \tag{2.10}$$

$$d^2S/dt^2 = 2DS^{-5} \tag{2.11}$$

The spherical space solution [$k = +1$ in equation (2.9)] represents a pulsating universe with an ordinary maximum at $S = D^{1/4}$ and a singularity at $S = 0$. The singularity corresponds to the breakdown of equation (2.3) for the

[6] See, e.g., Robertson & Noonan (1968), Chap. 17.

energy-momentum tensor as the density of matter gets high. The Euclidean space solution is given by

$$S(t) = 3D^{1/2}t^{1/3} \tag{2.12}$$

as compared to the $t^{2/3}$ behavior of the Friedmann universe. Equation (2.12), like the Friedmann universe, represents a contracting universe at $t < 0$ and an expanding universe at $t > 0$. The pseudospherical space solution [$k = -1$ in equation (2.9)] also represents a contracting universe at $t < 0$, and expanding one at $t > 0$.

The most significant difference between the cosmological models of equations (1.1) and the standard cosmological models of general relativity has to do with the time variation of all masses. For the standard models it follows from equations (2.4) that $\rho(t)S^3(t)$ is a constant and therefore mass is conserved. In the case of equation (1.1), however, $\rho(t)S^3(t)$ is a changing quantity. This can be understood to imply that all masses in the universe change at the same rate. The rate of change is obtained from equations (2.5) and (2.10):

$$\rho(t) = \rho_0 [S(t)]^{-6} \tag{2.13}$$

$$m(t) = m_0 [S(t)]^{-3} \tag{2.14}$$

$$\dot{m}(t)/m(t) = -3\dot{S}(t)/S(t) \tag{2.15}$$

Here $m(t)$ is the mass of a particle (or any other physical system held together by electromagnetic or strong interactions) at cosmic time t, and ρ_0 and m_0 are constants. Equations (2.8), (2.13), and (2.14) yield a simple relationship between mass and four-dimensional curvature:

$$m(t) = \tilde{m}_0 [K(t)]^{1/2} \tag{2.16}$$

where $\tilde{m}_0$ is a constant. *The relationship (2.16) is valid for all the isotropic, spatially homogeneous cosmological models of equation (1.1).*

Let us, finally, write down for future reference the expansion function for the cosmological model with positive constant four-dimensional curvature $K = b^{-2}$:

$$S(t) = b \sin(t/b) \tag{2.17}$$

This model[7] is not a solution of either equation (2.4) or equation (2.5). It represents a universe which starts with a singularity at $t = 0$, expands, and then contracts to a singularity at $t = \pi b$.

3. *Group Analysis of Cosmological Models*

A group analysis of space-times with constant four-dimensional curvature reveals a deep relationship between the geometry of space-time and fundamental properties of particles: The eigenvalues of the Casimir operators of the groups of motion can be interpreted in terms of masses and spins of particles. When the four-dimensional curvature K [equation (2.6)] is constant and positive the

[7] Robertson & Noonan (1968), Chap. 16.

group of motion is $O(3, 2)$, when $K = 0$ it is the Poincaré group, and when K is constant and negative the group of motion is $O(4, 1)$ (Wigner, 1939; Thomas, 1941).

Standard general-relativistic cosmological models, as well as the models discussed in Section 2, however, possess four-dimensional curvature that varies as a function of cosmic time; these models do not have a group of motion, and a straightforward application of group analysis is, therefore, impossible. Nevertheless, a relationship between the geometrical structure of the model and masses and spins of particles can be established along the following lines (Malin, 1974):

For any given isotropic spatially homogeneous universe U, at any given cosmic time t_0, let *an associated constant-curvature space-time* $M(t_0)$ be defined as that space-time, the constant (four-dimensional) curvature of which is the same as the four-dimensional curvature of U at time t_0. It is then postulated that *the masses and spins of particles in a universe U at time t_0 are given by the eigenvalues of the Casimir operators of the associated space-time $M(t_0)$.* The postulate is based on the following reasoning: In spite of the fact that space-time is *curved*, the Casimir operators of the Poincaré group, which is the group of motion of *flat* space-time, do correspond to masses and spins of particles. This amazing correspondence can be understood if the flat space-time of the Poincaré group is taken as a limiting case of curved space-time, and the Poincaré group representations are obtained from those of the groups $O(4,1)$ or $O(3,2)$ by the process of contraction (Segal, 1951; Inönu & Wigner, 1953; Inönu, 1965), as the curvature approaches zero. It seems, therefore, that the associated constant-curvature model $M(t_0)$ approximates U better than flat space-time, since the limit $K \to 0$ is not taken. In fact, since the curvature of $M(t_0)$ is *equal* to the curvature of U at time t_0 it is even possible that masses and spins in U will be given *exactly* by eigenvalues of the Casimir operators of $O(4,1)$ or $O(3,2)$, as the case may be.

The procedure outlined in the present section will be carried out now for the cosmological models of equation (1.1), the curvature of which is positive [equation (2.8)]. The group of motion of the associated space-times is $O(3,2)$.

4. *The Casimir Operators of the Group O(3,2) and the Poincaré Group*

The group $O(3,2)$, the group of motion in a space-time with constant positive four-dimensional curvature, is also the group of transformations in five-dimensional Euclidean space with coordinates $\xi_0, \xi_1, \xi_2, \xi_3, \xi_5$ that leaves invariant the hypersurface

$$\xi_0{}^2 - \xi_1{}^2 - \xi_2{}^2 - \xi_3{}^2 + \xi_5{}^2 = b^2 \tag{4.1}$$

Following Evans (1967), let us denote the infinitesimal operators of the group $O(3,2)$ by M_{ij} $(i, j = 0, 1, 2, 3, 5)$. They obey the commutation relations

$$[M_{ij}, M_{kl}] = g_{il}M_{jk} + g_{jk}M_{il} - g_{ik}M_{jl} - g_{jl}M_{ik} \tag{4.2}$$

where the metric tensor is defined by

$$g_{00} = g_{55} = 1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{ij} = 0 \quad (i \neq j) \tag{4.3}$$

The Casimir operators of the group $O(3,2)$ are

$$C_1 = \tfrac{1}{2} M_{ij} M^{ij} \tag{4.4}$$

$$C_2 = W_i W^i \tag{4.5}$$

where the W_i are defined as

$$W_i = \tfrac{1}{8} \epsilon_{ijklm} M^{jk} M^{lm} \tag{4.6}$$

(ϵ_{ijklm} is the totally antisymmetric tensor with five indices).

The Poincaré group is obtained from the group $O(3,2)$ by the process of contraction (evans, 1967), as $b \to \infty$. Let us define

$$\Pi_\mu = (i/b) M_{\mu_5}, \qquad \mu = 0, 1, 2, 3 \tag{4.7}$$

$$L_i = i M_{i_0}, \qquad i = 1, 2, 3 \tag{4.8}$$

$$J_i = \tfrac{1}{2} \epsilon_{ijk} M^{jk}, \qquad i, j, k = 1, 2, 3 \tag{4.9}$$

$$I_1 = -(1/2b^2) C_1 \tag{4.10}$$

$$I_2 = -(1/b^2) C_2 \tag{4.11}$$

$$P_\mu = \lim_{b \to \infty} \Pi_\mu \tag{4.12}$$

It follows from equation (4.2) that in the limit $b \to \infty$ P_μ, L_i, J_i satisfy the commutation relations of the infinitesimal generators of the Poincaré group; and, furthermore

$$\lim_{b \to \infty} I_1 = P_\mu P^\mu = m^2 \tag{4.13}$$

where m is the mass and

$$\lim_{b \to \infty} I_2 = m^2 s(s+1) \tag{4.14}$$

where s corresponds to the spin for a system with nonvanishing mass.

Because of equations (4.8)-(4.14) the Poincaré group is a valid approximation. In the physical world, however, the limit $b \to \infty$ is not taken and masses and spins should be obtained using the group $O(3,2)$ rather than the Poincaré group. This is done in the next section. Let us point out here that the structure of quantum theory has been developed for a universe with constant positive curvature (Dirac, 1935; Fronsdal, 1965), and difficulties that arise in the case of the group $O(4,1)$ do not exist for $O(3,2)$. In particular, in the case of $O(4,1)$ the concept of energy is problematic, because the spectrum of the Hamiltonian of all the irreducible representations is indefinite.[8] In the case of $O(3,2)$, how-

[8] This problem was discussed in detail by Philips & Wigner (1968).

ever, there are irreducible representations for which the spectrum of the Hamiltonian has a minimum and a casual structure was shown to exist (Fronsdal, 1965, 1973).

5. *Mass Variation*

Consider an isotropic, spatially homogeneous universe which is a solution of equation (1.2). Its associated constant-curvature space-time at cosmic time t_0 (see Section 2) is given by

$$M(t_0)\colon S(t) = b_0 \sin(t/b_0) \tag{5.1}$$

where, by equation (2.8)

$$b_0 = [K(t_0)]^{-1/2} = [\tfrac{1}{3}\pi\kappa\rho(t_0)]^{-1/2} \tag{5.2}$$

Let the mass and spin of a particle[9] in U correspond, at time t_0 to the eigenvalues $m_0{}^2$ and $m_0{}^2 s_0(s_0+1)$ of the operators I_1 and I_2 [equations (4.10) and (4.11)], or equivalently, to the eigenvalues

$$c_1{}^0 = 2b_0{}^2 m_0{}^2 \tag{5.3}$$

$$c_2{}^0 = b_0{}^2 m_0{}^2 s_0(s_0+1) \tag{5.4}$$

of the operators C_1 and C_2 [equations (4.4) and (4.5)]. As shown in a previous article (Malin, 1974) considerations of continuity and dimensionality lead to the conclusion that as a particle traces a world line in the universe U the eigenvalues of the Casimir operators C_1 and C_2 corresponding to the associated space-times do not vary as a function of cosmic time.

By equations (5.2), (5.3), and (5.4), if the constant eigenvalues of C_1 and C_2 associated with a particle are denoted by c_1 and c_2 then its spin $2c_1^{-1}c_2$ is a constant, but its mass varies as a function of cosmic time according to the formula

$$m(t) = (c_1/2)^{-1/2}[b(t)]^{-1} = [c_1 K(t)/2]^{1/2} \tag{5.5}$$

Comparing equation (5.5) with equation (2.16) we realize that *for the cosmological models of equation (1.1), the mass variation derived analytically from equation (2.5) is identical with the mass variation derived through the general group theoretical procedure.* These cosmological models can serve, therefore, as a theoretical testing ground for the consequences of the group analysis.

6. *A Contracting Universe*

In the present section the cosmological red-shift will be derived for the cosmological models of equation (1.1). The mass variation causes atomic fre-

[9] The term "particle" is used for brevity. Similar considerations apply to all physical systems held together by electromagnetic or strong interactions.

quencies to change; this change will be shown to be greater than the red or blue shifts of free photons in expanding or contracting universes. The net result is that cosmological *red*-shifts imply a *contracting universe.* It will be pointed out that such a contracting universe is not precluded by present observational evidence.

We assume a time variation of all masses according to equation (2.14), conservation of angular momentum, which is a rigorous consequence of the group approach [see Section 3 and Malin (1974)], as well as constancy of the fundamental constants c (the speed of light), e (the electron charge), h (Planck's constant), and κ (the constant of gravitation). Within the context of the present approach there is no reason to doubt the constancy of c, e, h, or κ.

For all atomic systems, held together by electromagnetic interaction, the following results are obtained by dimensional analysis, or else by a straightforward elementary calculation, and equation (2.14):

$$r(t) \sim [m(t)]^{-1} \sim S^3(t) \tag{6.1}$$

$$\nu(t) \sim m(t) \sim [S(t)]^{-3} \tag{6.2}$$

Here $r(t)$ is the radius of any atomic orbit, such as Bohr's radius, and $\nu(t)$ is any spectroscopic frequency.

For the following analysis we need some standard formulas (Weinberg, 1972b), the derivation of which is uneffected by the mass variation:

Let $\nu_1 = \nu(t_1)$ be the frequency of a photon emitted during an atomic transition at time t_1 at a distant galaxy with Robertson-Walker coordinates (t_1, r_1, θ, ϕ); let ν_0' be the frequency of the photon as observed here at the present time t_0; and, finally, let $\nu_0 = \nu(t_0)$ be the frequency of the same atomic transition at the present time t_0. Then

$$\nu_0'/\nu_1 = S(t_1)/S(t_0) \tag{6.3}$$

Also, defining

$$H_0' = \dot{S}(t_0)/S(t_0) \tag{6.4}$$

$$q_0' = -\ddot{S}\,(t_0)S(t_0)/\dot{S}^2(t_0) \tag{6.5}$$

one obtains

$$S(t_1) = S(t_0)[1 + H_0'(t_1 - t_0) - \tfrac{1}{2}q_0{}'H_0'{}^2(t - t_0)^2 + \cdots] \tag{6.6}$$

When the photon is observed having frequency ν_0', the red-shift is obtained by comparing ν_0' not with ν_1, but rather with ν_0, because a spectral line of frequency ν_1 at time t_1 has a changed frequency ν_0 at time t_0 [equation (6.2)]:

$$\nu_0/\nu_1 = [S(t_1)/S(t_0)]^3 \tag{6.7}$$

The observed red-shift is given, therefore, by

$$z = (\nu_0 - \nu_0')/\nu_0'$$

and because of equations (6.3), (6.6), and (6.7)

$$\begin{aligned} z &= [S(t_1)/S(t_0)]^2 - 1 \\ &= 2H_0'(t_1 - t_0) + (1 - q_0)H_0'^2(t - t_0)^2 + \cdots \end{aligned} \tag{6.8}$$

Following the steps of standard derivations (Weinberg, 1972b) the following relationships are now obtained:

$$t_1 - t_0 = (1/2H_0')[z - \tfrac{1}{4}(1 - q_0)z^2 + \cdots] \tag{6.9}$$

$$r_1 = (1/2S(t_0)H_0')[-z + \tfrac{1}{4}(2 - q_0')z^2 + \cdots] \tag{6.10}$$

$$\begin{aligned} d_L &\equiv L^{1/2}/4\pi l = (1/2H_0')[-z - \tfrac{1}{4}(q_0 - 2)z^2 + \cdots] \\ &= 10^{1+(m-M)/5} \text{ parsecs} \end{aligned} \tag{6.11}$$

d_L is the luminosity distance, L is the absolute luminosity, M is the absolute magnitude, and m is the apparent magnitude.

$$\begin{aligned} m - M &= 25 - 5 \log_{10}(-2H_0')(\text{km/sec/Mpc}) \\ &\quad + 5 \log_{10} cz(\text{km/sec}) + 1.086(1 + \tfrac{1}{2}q_0')z + \cdots \end{aligned} \tag{6.12}$$

It follows from equation (6.12) that the Hubble constant H_0, which is derived from the Hubble diagram, is related to the rate of change of the expansion function $S(t)$ by

$$H_0 = -2H_0' = -2\dot{S}(t_0)/S(t_0) \tag{6.13}$$

The observed red-shift corresponds, therefore, in the context of the cosmological models of equation (1.1), to a universe which is contracting at a rate corresponding to half the absolute value of Hubble's constant.

How can the value of the cosmological parameter q_0' be determined? Within the context of standard cosmological models determinations of q_0' on the basis of Hubble'a diagram are still quite uncertain (Sandage, 1972; Peach, 1970), especially when uncertainties in the course of galactic evolution over billions of years are taken into account. Estimates for q_0' are also available from estimates of the mean density of matter $\rho_0 = \rho(t_0)$. For the standard cosmological models one obtains from equation (2.4)

$$\rho_c = 3H_0^2/8\pi\kappa \tag{6.14}$$

$$q_0 = \rho_0/2\rho_c = 8\pi\kappa\rho_0/3H_0^2 \tag{6.15}$$

where ρ_c is "the critical density": if $\rho_0 > \rho_c$ and $q_0 > \frac{1}{2}$ the universe is closed, otherwise it is open. The analogous relationships for the cosmological models of equation (1.1) are obtained from equations (2.5) and (6.13):

$$\rho_c = 3H_0'^2/2\pi\kappa = 3H_0^2/8\pi\kappa \tag{6.16}$$

$$q_0' = 2\rho_0/\rho_c = 8\pi\kappa\rho_0/3H_0^2 \tag{6.17}$$

$\rho_0 = \rho_c$ corresponds now to $q_0 = 2$, *but the equation for* ρ_c *in terms of* H_0 *is the same as equation (6.14).* Current estimates of ρ_0 were recently reviewed by Gott III *et al.* (1974). They present a strong case for an open universe, conceding, however, that the evidence is not conclusive. In fact, possibilities of the universe being permeated by, e.g., relativistic gas, very low-mass particles, or ionized gas seem viable and are currently under active research (Rees, 1972; Coswik & McLelland, 1973). The reason for belief in and search for the so-called "missing mass" is clear: As pointed out by Einstein (1950), a closed universe is epistemologically much simpler and more elegant than an open one.

If equation (1.2) is valid, it follows from equation (6.13) that the universe is going now through a contracting phase. This possibility, for an open as well as a closed universe, is quite compatible with the strong existing evidence for "big-bang".[10] A full discussion of this point is contained in a forthcoming paper by V. N. Mansfield and S. Malin.

7. *Predictions and Evidence*

Numerical estimates for the rate of change of all masses are based on equations (2.15), (6.4), and (6.13). Current determinations for the value of Hubble's constant range between 40 and 60 km/sec/Mpc (Sandage, 1974; Sandage & Tammann, 1974; Branch & Patchett, 1973; Kirshner & Kwan, 1974; see also McVittie, 1974). When the uncertainties in these determinations are taken into account one ends up with the following estimate for mass variation:

$$\dot{m}/m = (+8 \pm 4) \times 10^{-11}/\mathrm{yr} \tag{7.1}$$

An analysis of the experimental and observational evidence for a mass variation of that order of magnitude was presented elsewhere (Malin, 1974). It was largely based on an article by F. J. Dyson (1972) concerning evidence for time variation of fundamental constants, and led to the conclusion that the wealth of available data do not preclude such a mass variation. The present section will deal with the effect of mass variation on dimensions and periods of orbits; this kind of evidence is the least ambiguous, and some of the relevant experiments are expected to be accurate enough to detect such an effect, if it exists, within the next few years.

From Newton's law of gravitation and conservation of angular momentum one obtains

$$\dot{r}/r = -3\dot{m}/m \tag{7.2}$$

$$\dot{T}/T = -5\dot{m}/m \tag{7.3}$$

where r is the radius of an orbit and T the period of revolution. For an atomic clock, however, the frequency ν is proportional to m, [equation (6.2)] and therefore,

$$\dot{\nu}/\nu = \dot{m}/m \tag{7.4}$$

[10] It is perhaps worthwhile noting that the evidence for a "big-bang", impressive as it is, is not universally accepted as conclusive (Burbidge, 1971).

(See also Gasiorowicz, 1974). When interplanetary ranging experiments and planetary orbit observations are carried out using an atomic clock as standard of time one obtains

$$(1/r)(dr/d\tau) = -2\dot{m}/m = (-16 \pm 8) \times 10^{-11}/\text{yr} \tag{7.5}$$

$$(1/T)(dT/d\tau) = -4\dot{m}/m = (-32 \pm 16) \times 10^{-11}/\text{yr} \tag{7.6}$$

where τ is atomic clock time. Equations (7.5), (7.6) are first order in $\dot{m}/m$ (an obviously adequate approximation).

Experimental work for the detection of such small effects was done in a different theoretical context, that of a possible time variation of the cosmological constant κ (Dirac, 1937 and 1938). In the case of gravitational orbits the effect of mass variations given by equation (7.1) is the same as that of an unchanging mass along with variations in κ given by[11]

$$\dot{\kappa}/\kappa = (+16 \pm 8) \times 10^{-11}/\text{yr} \tag{7.7}$$

This result provides a clear-cut distinction between the present theory and Dirac's hypothesis as well as other current theories with a changing κ: Equation (7.7) predicts an *increasing* κ, while Dirac's hypothesis predicts a *decreasing* κ.

Experimentally there seems to be no conclusive evidence for either a decrease or increase in κ of the order of magnitude of equation (7.7) (Dyson, 1972). Van Flandern (1975) has recently analyzed lunar occultation observations and obtained the value of $(-27'' \pm 18'')/\text{cy}^2$ for the anomalous acceleration of the Moon's mean longitude. He suggests, as a plausible interpretation, that κ changes at the rate of $\dot{\kappa}/\kappa = (-8 \pm 5) \times 10^{-11}/\text{yr}$. Dyson (1972) points out, however, the theoretical uncertainties in lunar orbit determinations and concludes that "the Moon's motion will probably remain too much affected by theoretical ambiguities to be a decisive test" for Dirac's hypothesis.[12] He continues his analysis by pointing out that in the long run interplanetary ranging observations will provide a better way of testing such variations in κ, because "the planets Venus, Mercury, and Mars describe orbits which are undisturbed by tidal effects to the precision here required." Such experiments are being conducted over the last few years by Shapiro *et al.* (1971). Shapiro's most recent result so far is (Shapiro, 1974)

$$\dot{\kappa}/\kappa = (+4 \pm 8) \times 10^{-11}/\text{yr} \tag{7.8}$$

There is every reason to believe, therefore, that conclusive evidence will be available within the next few years.

11 This equivalence holds only for gravitational orbits. Other theoretical predictions based on mass variation are quite different from the results of a changing κ (Malin, 1974).

12 Dirac's hypothesis is $\kappa \sim t^{-1}$, where t is the age of the universe; the numerical value of $\dot{\kappa}/\kappa$ according to Dirac's hypothesis is of opposite sign and somewhat smaller in magnitude than equation (6.7).

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Appendix

When mass variations according to equation (2.14) are considered, one may ask what happens to other physical quantities, such as momentum, energy, etc. The answer turns out to be fairly simple. It was shown by Schrödinger (1939) that momenta of massive as well as massless particles are changing according to

$$p(t) \sim [S(t)]^{-1} \tag{A1}$$

The variation of other quantities is simply derived from equations (2.14) and (A1).

Examples: The formula $E^2 = m_0{}^2c^4 + p^2c^2$ uniquely determines the variation of the energy E; the formula $v = pc^2/E$ determines the velocity v; etc.

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